

GRE Subject: Mathematics  
Select Tips and Tricks

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Typset by Hossien Sahebame

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# 1 Number Theory

In this section, for any  $a, b \in \mathbb{Z}$ , let  $(a, b), [a, b]$  denote their *gcf* and *lcm* respectively.

- **Divisibility** For any integer  $n \ni 1 \leq n \leq a$ , the number of  $n$  such that  $y \mid n$  but  $x \nmid n$  is given by  $(y > x)$

$$\left(\frac{a}{y}\right) - \left(\frac{a}{[x, y]}\right).$$

- **Successor existence** For  $n \in \mathbb{Z}^+$ , if

$$1 \leq x_1 < x_2 < \dots < x_{n+1} \leq 2n,$$

then there is an  $i$  such that

$$x_{i+1} = x_i + 1.$$

- **Decimal Expansion** Recall:  $n!$  has  $\sum_{k \in \mathbb{N}} \lfloor \frac{n}{5^k} \rfloor$  zeroes in its decimal expansion
- **Partitions** If  $n$  is an integer, number of ordered partitions is  $2^{n-1}$ .
- **Number of divisors** Let  $\sigma(n)$  denote the number of divisors of  $n \in \mathbb{Z}^+$ . If we take  $n = \prod_{i=1}^k p_i^{q_i}$  as the unique prime factorization of  $n$ , then

$$\sigma(n) = \prod_{i=1}^k p_i^{q_i-1},$$

moreover,  $\sigma$  is multiplicative, that is,

$$\sigma(ab) = \sigma(a)\sigma(b).$$

- **Fun Example** If  $n = 2^n$  then the sum of divisors is given by

$$\frac{2^{n+1} - 1}{2 - 1}.$$

- **Fun Example** If  $P$  is product of any 4 consecutive integers, then  $P + 1$  is a perfect square.
- **Fun Example** If  $6x + 5y \equiv_{13} 0$ , for what  $b$  is  $8x + by \equiv_{13} 0$ :  
Find  $[8, 6] = 24$  to cancel out the  $x$ 's then you get

$$20y - 3by \equiv_{13} 0$$

in other words

$$7y - 3by \equiv_{13} 0$$

Must find  $b$  to satisfy

$$7 - 3b \equiv_{13} 0$$

so  $b = 11$ .

- **Fun Example** Greatest integer that divides  $p^4 - 1$  for prime  $> 5$ :

Write

$$p^4 - 1 = (p + 1)(p - 1)(p^2 + 1).$$

by the difference of 4ths and squares. Then as  $p > 5$ , for  $(p \pm 1)$ , one is divisible by 2, and the other by 4 thus

$$2 \cdot 4 = 8 \mid (p + 1)(p - 1).$$

Moreover  $p^2 + 1$  is divisible by 2 thus we have that

$$16 \mid (p + 1)(p - 1)(p^2 + 1).$$

For any  $p \neq 3$  we have that  $p \equiv 1 \pmod{3}$  then we have  $p^2 - 1 \equiv 0 \pmod{3}$  thus

$$3 \cdot 16 = 48 \mid (p + 1)(p - 1)(p^2 + 1)$$

Lastly, note all odd squares end in 1 or 9 mod 10 thus  $(p^2 + 1)$  or  $(p^2 - 1)$  is divisible by 5 and thus

$$3 \cdot 5 \cdot 16 \mid p^4 - 1.$$

is the integer in question.

- **Fermats Little theorem** For any prime  $p$  and integer  $a$ , we have

$$a^p \equiv a \pmod{p}$$

And if  $p \nmid a$  then

$$a^{p-1} \equiv 1 \pmod{p}$$

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## 2 Elementary Algebra/Geometry

- **Min & Max** For all  $x, y \in \mathbb{R}$ , we have

$$\max\{x, y\} = \frac{x + y + |x - y|}{2}.$$

and

$$\min\{x, y\} = \frac{x + y - |x - y|}{2}.$$

- **Binomial Theorem** For any binomial  $(x+y)^n$  we have by the Binomial theorem that

$$\begin{aligned}(x + y)^n &= \sum_{k=0}^n \binom{n}{k} x^{n-k} y^k \\ &= \sum_{k=0}^n \binom{n}{k} x^k y^{n-k}.\end{aligned}$$

Where  $\binom{n}{k}$  is the binomial coefficient given by

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}.$$

- **Distance between point line** If  $y = mx + b$  is a line and  $(p, q)$  is a point, then the shortest distance is given by

$$d = \sqrt{\frac{(b + mp - q)^2}{m^2 + 1}}.$$

- **Cubic Discriminant** If  $ax^3 + bx^2 + cx + d = 0$  is a cubic, then its discriminant is given by

$$\Delta = b^2c^2 - 4ac^3 - 4b^3d - 27a^2d^2 + 18abcd.$$

- **Euler Formulas** Recall by Euler we have that

$$\sin \theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}.$$

and

$$\cos \theta = \frac{e^{i\theta} + e^{-i\theta}}{2i}.$$

- An equation of the form

$$a \sin kx + b \cos kx$$

has max value

$$\sqrt{a^2 + b^2}.$$

- If the sum of coefficients of a polynomial is zero, then 1 is a root. I.e.,

$$x^3 - 3x^2 + 5x - 3$$

has 1 as a root.

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### 3 Calculus I-III

- **Integrals you should know!**

1.  $\int \sin x \cos x dx = \frac{\sin^2 x}{2} + C.$
2.  $\int \tan x dx = -\ln |\cos x| + C.$
3.  $\int \cos^2 x dx = \frac{x}{2} + \frac{\sin(2x)}{4} + C.$
4.  $\int \sin^2 x dx = \frac{x}{2} - \frac{\sin(2x)}{4} + C.$
5.  $\int \frac{1}{x \log x} dx = \log |\log x| + C.$
6.  $\int e^x \sin x dx = \frac{e^x}{2}(\sin x - \cos x) + C.$
7.  $\int e^x \cos x dx = \frac{e^x}{2}(\sin x + \cos x) + C.$
8.  $\int \frac{\cos x}{\cos x + \sin x} dx = \frac{x}{2} + \frac{\ln |\cos x + \sin x|}{2} + C.$
9.  $\int \sin^3 x dx = \frac{\cos^3 x}{3} - \cos x + C.$
10.  $\int \cos^3 x dx = -\frac{\sin^3 x}{3} + \sin x + C.$
11.  $\int_a^b f(x) dx$  can be written as

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \frac{b-a}{n} \sum_{k=1}^n f\left(a + k \cdot \frac{b-a}{n}\right)$$

Where  $\Delta x = \frac{(b-a)}{n}$ .

12.  $\int \frac{1}{1-\cos x} dx = -\cot\left(\frac{x}{2}\right) + C.$
  13.  $\int \tan x dx = -\ln(|\cos x|) + C$
- **$f$  VS.  $f'$**

1. If  $f(x)$  is increasing in an interval  $I$ , then  $f'(x) > 0$  on  $I$ .
2. Similarly, if  $f(x)$  is decreasing on an interval  $I$ , then  $f'(x) < 0$  on  $I$ .

- **Tangent Plane If**

$$z = f(x, y),$$

then the equation of the plane tangent at  $(x_0, y_0)$  is given by

$$z - f((x_0, y_0)) = z_x((x_0, y_0))(x - x_0) + z_y((x_0, y_0))(y - y_0).$$

- $\frac{d}{dx}$  of  $|\cdot|$  For the derivative of  $|f(x)|$ , using the square root of the square we have the derivative as

$$f'(x) \frac{f(x)}{|f(x)|}.$$

- **Piece-wise Derivative** If  $f$  is a function over closed unit interval given as the piece-wise

$$f(x) = \begin{cases} a^2 & x \in [0, a] \\ ax & \text{otherwise} \end{cases}$$

Then to determine the value of  $a \in [0, 1]$  such that

$$\int_0^1 f(x) dx = 1,$$

Split up the integral as

$$\int_0^a a^2 dx + \int_a^1 ax dx = 1$$

and solve for  $a$ .

- **Counter-Examples** Function that is continuous but not uniform

$$f(x) = \frac{1}{x}.$$

Function uniform not Lipschitz

$$f(x) = \sqrt{x}.$$

Function Lipschitz not differentiable is

$$f(x) = |x|.$$

- **FTC** By F.T.C.,

$$\frac{d}{dx} \int_0^{x^2} e^{-t^2} dt = 2xe^{-(x^2)^2}$$

Where the  $2x$  gets tagged on by chain rule.

- **Taylor series** For a given function  $f(x)$ , we can approximate it using the Taylor series expansion

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)(x-a)^n}{n!}.$$

- **Generalized Derivatives**

1. If  $f(x) = e^{kx}$  then the generalized  $n$ th derivative is given via

$$f^{(n)}(x) = (k)^{n-1} e^{kx} (kx + n).$$



2. If  $f(x) = x \sin x$  then the  $n$ th derivative is given via

$$f^{(n)}(x) = n \sin\left(x + \frac{(n+1)\pi}{2}\right) + x \sin\left(x + \frac{n\pi}{2}\right)$$

3. If  $f(x) = \sin x$ , then the  $n$ th derivative is given by

$$f^n(x) = n \sin\left(x + \frac{\pi n}{2}\right).$$

4. If  $f(x) = \cos x$ , then the  $n$ th derivative is given by

$$f^n(x) = n \cos\left(x + \frac{\pi n}{2}\right).$$

- **Point & Plane** If  $Ax + By + Cz + D = 0$  is a plane and  $(x_0, y_0, z_0)$  is a point in 3-space, then the distance is given via

$$d = \frac{Ax_0 + By_0 + Cz_0 + D}{\sqrt{A^2 + B^2 + C^2 + D^2}}.$$

- **Plane & Plane** If  $Ax + By + Cz + D_1 = 0$ ,  $Ax + By + Cz + D_2 = 0$  are parallel planes, the shortest distance between them is given via

$$d = \frac{|D_2 - D_1|}{\sqrt{A^2 + B^2 + C^2}}.$$

- **Max Value** If  $f(x_1, \dots, x_n)$  is a function, the max value of the directional derivative at  $(a_1, \dots, a_n)$  is length of

$$\langle \partial f_{x_1}(a_1, \dots, a_n), \dots, \partial f_{x_n}(a_1, \dots, a_n) \rangle$$

- **Curve length** If  $x(t), y(t)$  for  $t \in [a, b]$  are parameterized curves, then their length is given via the definite integral

$$\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt,$$

Where  $\left(\frac{dx}{dt}\right), \left(\frac{dy}{dt}\right)$  are first partial derivatives of  $x$  and  $y$  respectively, with respect to  $t$ .

- **Fun Example** Note the series

$$\sum_{k=0}^{\infty} \frac{1}{k} - \frac{1}{2^k}$$

goes to  $\infty$ .

- **Fun Example** The series  $\sum_{k=1}^{\infty} \frac{k^2}{(k-1)!}$  converges to  $2e$ . To see this note

$$\begin{aligned}
 \sum_{k=1}^{\infty} \frac{k^2}{(k-1)!} &= \sum_{k=1}^{\infty} \frac{k}{(k-1)!} \\
 &= \sum_{k=1}^{\infty} \frac{(k-1) + 1}{(k-1)!} \\
 &= \sum_{k=2}^{\infty} \frac{1}{(k-2)!} + \sum_{k=1}^{\infty} \frac{1}{(k-1)!} \\
 &= e + e \\
 &= 2e.
 \end{aligned}$$

as needed.

- **Fun Example** If  $f$  and  $g$  are real-valued and differentiable on closed unit interval and  $f'(x) \geq g'(x)$ ,  $\forall x \in [0, 1]$ , then

$$f(1) - g(1) \geq f(0) - g(0).$$

This is by Mean value theorem.

- If  $(a_1, b_1, c_1), (a_2, b_2, c_2), (a_3, b_3, c_3)$  are points. The equation of the plane is:  
Take cross product of  $P_1P_2$  and  $P_2P_3$  then replace any point to find the constant.

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## 4 Analysis/Topology

- **Continuous Piece-Wise** For a piecewise function  $g(x)$  with  $x \in \mathbb{R}$ , defined by 1 for  $x \in \mathbb{Q}$  and  $e^x$  for  $x \in \mathbb{R} \setminus \mathbb{Q}$  continuous for  $e^x = 1$  thus only at  $x = 0$ .
- **LUB Property** If  $\emptyset \neq B \subset \mathbb{R}$  has a least upper bound  $b$  with  $b \notin B$ , then  $b$  is a limit point of  $B$ . (This is because  $\overline{B} = B \cup$  limit points of  $B$ )
- If  $f : \mathbb{R} \rightarrow \mathbb{R}$  is continuous, then the set

$$A = \{f(x) : x \in (a, b)\}$$

is both connected and bounded, in other words images of bounded AND connected are preserved under continuous functions.

- **Metric spaces** If  $d$  is a metric on a set  $S$ , then

$$d + k, e^d + k, d^2$$

are not metrics, however

$$\sqrt{d}$$

is.

- **Limit Point Sets** For any topological space  $X$  and subset  $Y$ , let  $Y'$  denote the set of limit points of  $Y$ . Then for any subsets  $A, B$  of  $X$  then

$$(A \cup B)' = A' \cup B',$$

and if  $A' = \emptyset$  then  $A$  is closed.

However, intersections are not preserved and if  $A \in \tau_X$ , then  $A'$  may or may not be empty.

- **Nested Invariants** If  $A \subset B \subset C$ , if  $A, C$  are connected, compact, or Hausdorff, then  $B$  is only guaranteed the Hausdorff property.
- **Lower Limit Topology** Facts about  $\mathbb{R}_l$ :
  1. Hausdorff
  2. Lindelof
  3. NON COMPACT
  4. TOTALLY DISCONNECTED.
- **K-Topology** Facts about  $\mathbb{R}_K$ :
  1. Connected but NOT path

- 2. Hausdorff
- 3. NON COMPACT

- **Cofinite Topology** Facts about  $\mathbb{R}_{\text{cofinite}}$ :

- 1. NON HAUSDORFF
- 2. COMPACT
- 3. infinite sets are NOT closed as their complements are NOT open.

- **Mobius Transformation** If  $f(z) = \frac{az+b}{cz+d}$  is a given mobius transformation, then the fixed points are given by setting

$$\frac{az + b}{cz + d} = z.$$

using quadratic to solve for  $z$ .

- **Continuity** In the  $\epsilon, \delta$  definition if

$$\lim_{x \rightarrow x_0} f(x) = L,$$

then the least upper bound on  $\delta$  is given by

$$\frac{\epsilon}{f'(c)}.$$

- **Fun Example** If a metric is defined on Riemann integrable functions

$$d(f, g) = \int_0^1 |f - g| dx,$$

then

$$d(f, g) > 0$$

is NOT satisfied if  $f \neq g$ . Simply define  $f$  to be 1 for every  $x \in [0, 1]$  and  $g$  to be 1 for all the  $x$  but equal 2 when  $x = 1$ .

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## 5 Abstract & Linear Algebra

- **Determinant Properties** For  $n \times n$  matrices  $A, B$  if  $|A|$  denotes determinant of  $A$ , then behold, properties of the determinant

1.  $|AB| = |A||B|$
2.  $|cA| = c^n|A|$
3.  $|A^{-1}| = \frac{1}{|A|}$
4.  $|A| = |A^T|$
5.  $|A| = \prod \lambda_i$  where the  $\lambda_i$  are eigen values of  $A$ .
6. Swapping rows negates determinant.
7. Scalar any row of  $A$  by  $k$  then new determinant  $= k|A|$ .
8. Given a  $3 \times 3$  matrix

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix},$$

We can compute the determinant via

$$\det A = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix}.$$

9. **(Relate det & inverse)** For  $n = 2$ , if  $A$  (non-singular) is given via

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix},$$

then its inverse is given via

$$A^{-1} = \frac{1}{|A|} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

- **Trace Properties**

1.  $\text{tr}A = \text{tr}(A^T)$
2.  $\text{tr}(A + B) = \text{tr}A + \text{tr}B$
3. Scalar comes out
4. NOT multiplicative

- **Dimension of Sum** For two subspaces of dim 10 space,

$$\dim(V_1 + V_2) \leq 10.$$

Also

$$\dim(V_1 + V_2) = \dim(V_1) + \dim(V_2) - \dim(V_1 \cap V_2).$$

So if  $V_i$  are both dim 6, intersection is 2. More generally, for any two vector space or subspace, say  $W, Z$ , one has

$$\dim(W + Z) = \dim W + \dim Z - \dim(W \cap Z).$$

- **Dimension of intersection w.r.t. ambient space** If  $U, V$  are subspaces of  $\mathbb{R}^n$ , then

$$\dim(U) + \dim(V) - \dim(U \cap V) \leq \dim(\mathbb{R}^n) = n$$

- **Eigen Values** For any given  $n \times n$  matrix  $A$ , the product of the eigen values is given by

$$\prod_{i=1}^n \lambda_i = \det(A - \lambda I).$$

And the sum of the eigen values is given by

$$\sum_{i=0}^n \lambda_i = \text{tr}(A).$$

- **Eigen Vectors** If  $V$  is vector space of continuous functions and  $D : V \rightarrow V$  sends  $f \rightarrow f'$ , what are eigen vectors of  $D$ ?

We need solutions to  $f'(x) = \lambda f(x)$ , so all nonzero function of the form

$$ke^{\lambda x}.$$

- **Non - Subring Example** If  $R$  is a polynomial ring of polynomials over  $\mathbb{R}$ . Then the set of even degree polynomials is not a subring as

$$x^2 + x, -x^2$$

are not closed under addition.

- **Subring Example** If  $A, B$  are subspace of vector space  $V$ , then  $A \cap B$  and  $A + B$  are subspaces not complement or union.
- Linear combo of rows does nothing.

- **Iso of  $\mathbb{Q}$**  Number of ring isomorphisms of  $\mathbb{Q}$  to itself (as a field) is only 1, trivial sending 1 to 1.
- **Similar Matrices** If  $A, B$  are similar, then they have same trace, they're inverses are similar, and so are  $A - 2I, B - 2I$ .
- **IDs & Irreducibles** If  $D$  is integral domain of form  $r + s\sqrt{17}$  then irreducibles are when

$$(r, s) = 1.$$

- **Ideals** Only ideals of  $\mathbb{Z}$  are of the form  $n\mathbb{Z}$  for some  $n \in \mathbb{Z}$ .
- **Einstein Criteria for Irreducibility** If  $P(x) = a_nx^n + a_{n-1}x^{n-1} + \dots + a_1x + a_0$ . then Einsteins criteria for irreducibility states if there exists a prime  $p$  such that

1.  $p|a_j; 0 \leq j < n$ .
2.  $p \nmid a_n$
3.  $p^2 \nmid a_0$ ,

Then  $P$  is irreducible.

- **Hom/Aut Group**

1. For the modular groups,

$$|\text{Hom}(\mathbb{Z}/m\mathbb{Z}, \mathbb{Z}/n\mathbb{Z})| = (m, n),$$

the *gcd* of  $m$  and  $n$ .

2. For a field  $F$  with multiplicative and additive map into itself, then

$$|\text{Aut}(F)| = 1.$$

- **Cauchy #** The *Cauchy Number* for a given element of the symmetric group is the sum of  $k_i - 1$  where  $k_i$  is the length of cycles.
- **Cycle length is the order** If  $\sigma \in S_n$ , then the order of  $\sigma$  is his own cycle length.
- If  $\sigma$  is a  $k$ -cycle and  $n \in \mathbb{N}$ , then

$$\sigma^n = \sigma^{n \bmod k}$$

- If  $U, W$  are subspaces of vector space  $V$ , so is  $nU, nW$  and  $U \cap W$  but NOT  $U \cup W$ .

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## 6 Set Theory & Logic

- If  $A = \{1, 2, \dots, k\}$  then the number of bijections from  $A$  onto itself is  $k!$ .
- The number of injection from  $\{1, 2, \dots, m\}$  to  $\{1, 2, \dots, n\}$  where  $m < n$  is equal to

$$(n)(n-1)(n-2) \cdot \dots \cdot (n-(m+1)).$$

- Given the statement

$$\forall x \in \mathbb{R}, \exists y \in \mathbb{R} \ni (f(x) > 0) \Rightarrow (g(y) > 0).$$

Then the negation is given by

$$\exists x \in \mathbb{R} \ni \forall y \in \mathbb{R}, (f(x) > 0) \wedge (g(y) \leq 0).$$

- Let  $A, B$  be subsets of  $M$  and  $S_0 = \{A, B\}$ . If  $S_{i+1}$  is defined by taking unions, intersections, and complements, how many elements are in  $\bigcup S_i$ ? 16.
- For any finite set  $X$  of  $n$  elements,

$$|\mathcal{P}(X)| = 2^n,$$

however in the case that intersection of all is nonempty, the cardinality is

$$2^{n-1}.$$

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## 7 Misc.

- Probability  $x \geq yz$  if  $x, yz \in [0, 1]$  is given via the iterated integral

$$\int_0^1 \int_0^1 (1 - yz) dy dz.$$

- Probability of arranging  $n$  distinct things is  $n!$  so probability of having one formation is  $\frac{1}{n!}$ .
- If  $x = \sqrt{n + \sqrt{n + \sqrt{n + \dots}}}$ , then  $x = \frac{1 + \sqrt{4n+1}}{2}$ .

- Probability of picking  $k$  correct answers out of  $n$  question each with  $x$  choices is

$$\binom{n}{k} \left(\frac{1}{x}\right)^k \left(\frac{x-1}{x}\right)^{n-k}.$$

- Let  $X$  be discrete random variable uniformly distributed with values

$$x_1, \dots, x_n.$$

Then the variance of  $X$  is the mean of the squares minus the square of the means.

- Number of ways of choosing  $m$  letters from  $n > m$  is  $n^m$  if you CAN repeat, if you cannot then its

$$(n)(n-1)\dots(n-k)$$

where  $n-k$  is the  $m$ th number down backwards, for example selecting 5 letter code from 7 letters if no repeating is

$$7 \cdot 6 \cdot 5 \cdot 4 \cdot 3.$$

Where 3 is the  $m$ th.

- If  $f(x), g(x)$  are solutions to linear homogeneous DE on  $(a, b)$  then both 0 and linear combos are solutions, not products.
- *Bernoulli's Equation* is given via

$$\frac{dy}{dx} + P(x)y = Q(x)y^n,$$

for  $n \geq 2$ .

- *Ricatti's Equation* is given via

$$\frac{dy}{dt} = q_1(t) + q_2(t)y + q_3(t)y^2,$$

for  $n \geq 2$ .

- If  $f(x)$  is a probability density function, the its standard deviation is given by

$$\sqrt{\int_a^b x^2 f(x) dx - [\int_a^b x f(x) dx]^2}$$

- If  $f(x) = \sin(nx)$  the Laplace transform is given via

$$\frac{a}{s^2 + a^2}.$$

- If  $f(x) = \cos(nx)$  the Laplace transform is given via

$$\frac{s}{a^2 + s^2}.$$

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## 8. Conic Sections

- Circle is given by

$$(x - h)^2 + (y - k)^2 = r^2$$

Where the center is  $(h, k)$  and radius is  $r$ .

- (Horizontal) Ellipse is given by

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1,$$

where

1.  $a < b$ .
  2. Major:  $(h \pm a, k)$  ; Minor  $(h, k \pm b)$ .
  3. Foci:  $(h \pm c, k)$  where  $c^2 = a^2 - b^2$ .
- (Vertical) Ellipse is given by

$$\frac{(x - h)^2}{b^2} + \frac{(y - k)^2}{a^2} = 1,$$

where

1.  $a < b$ .
  2. Major:  $(h, k \pm a)$  ; Minor  $(h \pm b, k)$ .
  3. Foci:  $(h, k \pm c)$  where  $c^2 = a^2 - b^2$ .
- (Transverse horizontal) Hyperbola is given by

$$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1.$$

where

- (Transverse vertical) Hyperbola is given by

$$\frac{(x - h)^2}{b^2} - \frac{(y - k)^2}{a^2} = 1.$$

where

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## Even & Odd

- A function,  $f(x)$ , is said to be *even* if

$$f(-x) = f(x),$$

for every  $x$  in its domain.

- A function,  $f(x)$ , is said to be *odd* if

$$f(-x) = -f(x),$$

for every  $x$  in its domain.

- Examples of even odd functions.

1.  $f(x) = \sin x$  is odd
2.  $f(x) = \cos x$  is even
3.  $f(x) = \tan x$  is odd
4.  $f(x) = e^x$  is even
5.  $f(x) = x^n$  is even for even  $n$  and odd for odd  $n$ .

- Rules of even odd functions

1. (odd)(even)=(odd)
2. (odd)(odd)=(even)
3. (even)(even)=(even)
4.  $\frac{(\text{even})}{(\text{even})}=(\text{even})$
5.  $\frac{(\text{odd})}{(\text{even})}=(\text{odd})$
6. (even) $\pm$ (even)=(even)
7. (odd) $\pm$ (odd)=(odd)
8. Sum of even and odd is neither unless one is identically zero.
9.  $\circ$  of odds is odd,  $\circ$  of evens is even.

- Note that

$$\int_{-a}^a f(x)dx = 0$$

when  $f(x)$  is odd and

$$\int_{-a}^a f(x)dx = 2 \int_0^a f(x)dx$$

when  $f(x)$  is even.

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