MTED 697 Project: An implicit and explicit analysis of my degree - CSULB

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Part §I.1 What & Why

In this paper my aim is to analyze my Pure Mathematics Masters Degree in both an implicit and an explicit sense i.e., the actual mathematical concepts covered in courses versus what I learned about myself as a learner of mathematics.

Often times I get so caught up in the courses and in the moment that it is easy to forget about taking a step back and looking at the bigger picture. The goal of this paper is to do just that. Each course typically includes several big ideas or "Theorems" which is what I will be covering in the next sections. I then move to the implicit analysis which is not really mathematical but rather I dig into struggles I overcame in the process, the highs and lows, and what I learned about myself as I learned this content.

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Part §I.2 The Courses

For this analysis I have selected courses that particularly stood out for me because I struggled more and grasped much more of the material. One additional concept I will cover is the notion of some of the courses overlapping in unique ways to form a new field of study in some sense.

The first course I will be covering is Elementary Number Theory which does not have much to do with the other courses but rather exposed me to the relationship numbers have with one another. In this course we went over modular arithmetic, prime divisors and relative prime numbers, and some of the major related theorems (e.g. Bezout's, Wilson's, and Fermat's Little).

Next I examine Point Set Topology which is one of the corner stone courses for any pure mathematician. This course goes over a generalized Euclidean space called metric spaces and furthermore to topological spaces where we no longer have this notion of distance but only open sets. A few concepts I will go over will be notions of continuity in terms of open sets and how it relates to the $\epsilon - \delta$ definition from calculus, the notion of closed and bounded and why closed intervals attain min and max points under continuity, connectedness, and compactness. Some key theorems and results will be Tychonoffs Theorem, Heine-Borel, and theorems related to invariants under Continuity.

The last one will be the first course of a two-parted Complex Analysis sequence I took. A first course in Complex Analysis is basically calculus over the complex numbers, or the analysis of real numbers extended to the complex plane. One major thing that changes is the fact that continuity over $\mathbb{R}$ is much nicer in a sense that we only have two directions to check for continuity, left and right. When we work over $\mathbb{C}$ however, continuity can hit from 360 degrees thus the simple left right continuity does not suffice. Additionally, the use of theorems from Real Analysis such as the Monotone Convergence Theorem fail as $\mathbb{C}$ is no longer totally ordered. Some of the big results and tools used for integration come from the Cauchy-Integral formula and we build to Liouville's Theorem, which states that bounded and everywhere continuous complex functions need be constant, which I thought was a strong statement! I will end the first part with the Max Modulus Principal, which takes advantage of results from the Point Set Topology Course.

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Part §II.1 Number Theory

In Number Theory I learned many techniques and general results which can be applied to the mathematics of everyday life, teaching mathematics as the K-12 levels, as well as in abstract algebra and other advanced pure mathematics courses. The course begins with some more foundational concepts generalized such as the greatest common divisor and the least common multiple of any two given integers then moves onto modular arithmetic and incorporates this into theorems about prime divisibilty.

Throughout grade school I have been taught things like long division and factors and multiples primarily without any rigour or reason as to why but more on the how. It was not until my course in Elementary Number Theory however that I learned to generalize these concepts even more and learned more of the why as apposed to merely the how. I was taught in elementary school how to find factors of two given integers and how to use these to determine their associated greatest common divisor (GCD) and least common multiple (LCM), in this course I learned the key relationship these all share: Given $a, b \in \mathbb{Z}$, if we denote G_{ab}, L_{ab} as their GCD and LCM respectively, then the following equality holds (and I learned to prove it!):

$$ab = G_{ab}L_{ab}$$

This is a fact I never knew until this course and it goes without saying that this helps me when helping other students with these concepts. One immediate result is that if two integers are coprime to each other, meaning the only common factor they share is 1, then their LCM is automatically their product. Furthermore one can divide the product by the GCF to obtain the LCM.

Additionally, given any two distinct integers a, b where one of them, where a is greater than zero, one can always find unique integers q, r where $0 \leq r < a$ such that

$$b = aq + r.$$

This is known as the Division Algorithm, which is really a generalization of the long division I learned in elementary school. Not only did I learn this generalization but I also saw how to prove this fact first by proving existence of q, r then proving their uniqueness.

In K-12 schooling I learned about some basic divisibility rules such as 2,5, and 10. That is, given any integer one can quickly determine if the given integer is divisible by 2, 5, or 10. It turns out using a bit of modular arithmetic one can figure out and prove the rules of divisibilty for 3,7, and 11:

Let a given integer n have digit expansion

$$n = a_k a_{k-1} ... a_0.$$

For example the for the integer 269 we have that $k = 2$ where

$$a_0 = 9$$

$$a_1 = 6$$

$$a_2 = 2,$$

And so divisibility for 3 states that if

$$3 \Big| \sum_{i=0}^{k} a_i$$

then

$$3 \Big| n.$$

Thus for the integer $n = 269$ as 3 does not divide $2 + 6 + 9$, 3 does not divide n either. For the 11 rule we have that if

$$11 \Big| \sum_{i=0}^{k} (-1)^i a_i$$

Then

$$11 \Big| n.$$

And lastly for the 7 rule we have that if

$$7 \Big| a_k a_{k-1} ... a_1 - 2a_0,$$

then

$$7 \Big| n.$$

Using these together with the following result about divisibility we can obtain so many more rules as well:

If a, b are coprime and both $a|c$ and $b|c$, then $ab|c$.

For example, $3 \mid 30$ and $5 \mid 30$ and since $3, 5$ are coprime, then we can conclude that $(3 \cdot 5 = 15) \mid 30$. However, if we take 3 and 6 it does not work, as 3 and 6 are not coprime and clearly $(3 \cdot 6 = 18) \nmid 30$.

Going a bit further into the course I learned about the Fundamental Theorem of Arithmetic which states:

Any integer $n > 1$ has a unique factorization into primes, up to order of factors.

That is, for any integer $n > 1$, we can uniquely write

$$n = p_1^{\alpha_1} \cdot p_2^{\alpha_2} \cdot ... \cdot p_k^{\alpha_k},$$

where the p_i are distinct primes and the α_i are not necessarily distinct positive integers. This key fact is used in proving basics about primes and divisibility up to big theorems in finite group and ring theory as the orders of groups/rings can be expressed in this fashion.

This leads me to the last topic which was Euler's Totent Phi function, Fermat's Little Theorem/Wilson's Theorem. So Fermat's Little Theorem states:

If p is prime, then for any integer a we have that $p \Big| a^p - a$, which is actually a special case of Euler's Theorem, which states that for any modulus n and a relatively prime integer say a, we have that

$$n \Big| a^{\phi(n)} - 1,$$

where $\phi(n)$ is the number of relatively prime integers to n. These results helped me see the bigger picture of the set of integers and the structure between the elements, i.e., the way numbers relate to one another via divisibility and prime decompositions much like studying the English alphabet in order to better understand our language. Much of these results have been centered around some sort of primal testing which takes us to the last result covered, Wilson's Theorem, which states:

$$\textit{A natural number } n > 1 \textit{ is prime if and only if } n \Big| (n-1)! + 1 \ ,$$

which is a way of testing for primality. Much of these results rely heavily on modular arithmetic, which I had never mastered in my undergrad abstract algebra course and never fully understood until I took number theory. In the bigger picture, modular arithmetic is a way of relating numbers to one another via a common divisor to form some partition of the whole set of integers into equivalence classes. This was something I struggled with heavily for some odd reason but was able to finally wrap my head around the concept by the end of this course.

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Part §II.2 Topology

Going into Point Set Topology I knew a bit about generalized metric spaces and not so much about generalized topological spaces and all the different topologies one can place on a space/set. I had a naive intuition that things from $\mathbb{R}$ just carry on over. However, when generalizing from a metric space to a topological space, we lost this notion of "distance" between two points in a space. We lose this notion of smallness that continuity tends to preserve. All we have to work with is a union of open subsets of our given space that actually covers the space and in fact the notion of a "continuous function" where distance is typically preserved gets generalized to a "continuous function" where "openness" is preserved with the map and in fact in any metric space these two notions become the same!

Given a set X, a *topology* on X, often denoted $\mathcal{T}$, is a set of subsets of X such that

$$\text{(i) } \emptyset, X \in \mathcal{T}$$

$$\text{(ii) If } U_\alpha \in \mathcal{T},\ \forall \alpha \in A \text{ any indexing set, then } \bigcup_{\alpha \in A} U_\alpha \in \mathcal{T}$$

$$\text{(iii) If } U_1, ..., U_N \in \mathcal{T} \text{ for some } N \in \mathbb{N}, \text{ then } \bigcap_{i=1}^{N} U_i \in \mathcal{T}$$

and if you recall from Linear Algebra, every vector space has a basis which spans the entire space. Analogously in topology one has a *base for a topology* which is a collection of subsets of X denoted $\mathcal{B}$ such that any subset of X can be written as a union of elements in $\mathcal{B}$.

Some examples of a basis can be seen if we consider out topological space to be $X = \mathbb{R}$ or $X = \mathbb{R}^2$. First take $X = \mathbb{R}$, then a basis for the topology on $\mathbb{R}$ can be given as

$$\mathcal{B} := \{(a, b) \subseteq \mathbb{R} \mid a < b\}$$

which is basically just the open intervals of $\mathbb{R}$. If we let $X = \mathbb{R}^2$, then we could take

$$\mathcal{B} := \{B_\epsilon(x) \mid x \in \mathbb{R}^2, \epsilon > 0\}$$

to be a basis on $\mathbb{R}^2$ which is really just a collection of open epsilon balls. One big upshot here is that now to determine whether or not a given function is continuous, instead of checking if distance is "preserved" (that is, two points in the domain within δ distance of one another guarantees their images are within ϵ distance) one checks if open sets in the codomain have preimages that are open in the domain, or more formally,

A function $f : X \to Y$ is continuous if $f^{-1}(U)$ is open in X whenever U is open in Y.

And it turns out that this definition of continuity agrees with the $\epsilon - \delta$ definition when we take the spaces to both be $\mathbb{R}$. Furthermore, this notion of continuity can be extended similar to how we can extend the notion of homomorphism in abstract algebra. That is, a continuous function that is 1-1 and onto as well is called a *homeomorphism* which in fact is what determines the notion of equivalence for topological spaces. So then two topological spaces X, Y are said to be equivalent if and only if there exists a homeomorphism from one to the other.

Often times in mathematical analysis we are concerned with properties that carry over under a given continuous function. Some of these properties, such as connectedness and compactness, have been made rigorous via topology. This is one reason I feel like these courses should be taught simultaneously.

One of the two properties I will go over will be this notion of *connectedness* of a given space. First we will need some definitions. Note that a separation of a space X is a union of disjoint non-empty open subsets of X in which neither of the sets is all of X, that is, we can write

$$X = A \cup B$$

where $A, B \subsetneq X$ are nonempty, open, and

$$A \cap B = \emptyset.$$

Then we say a given topological space X is *connected* if there does not exists a separation of the space. And so spaces that have a separation we refer to as a *disconnected*. Some familiar examples of connected spaces we have all seen can be $\mathbb{R}$ or intervals in $\mathbb{R}$. Given real numbers a, b, c such that $a < b < c$, one trivial example of a *disconnected* space would be say

$$(a, b) \cup \{c\}.$$

In this setting then, one intuitive question becomes "is the image of a connected space under a continuous function always connected?" and the answer is YES! thus

Let $f : X \to Y$ be continuous. If X is connected, then so is $f(X)$.

One big result that relies heavily on this fact is from complex analysis which states differentiable functions defined on open subsets of $\mathbb{C}$ that have a 0 derivative need be constant given the domain is a connected space. So if we take the domain to be all of $\mathbb{C}$ (as $\mathbb{C}$ is connected), then any complex-differentiable function whose derivative is 0 everywhere need be the constant function. Anytime a property is preserved under a continuous function we refer to the property as *topological*.

Another topological property that we often use without even knowing is compactness. In order to define compactness we must first define what is means to be an open cover. We say a collection of sets $\{U_\alpha\}_{\alpha \in A}$ is an open cover for a given space X if $U_\alpha \subseteq X$ is open for each $\alpha \in A$ and

$$X \subseteq \bigcup_{\alpha \in A} U_\alpha.$$

Then we can say a give topological space is considered to be *compact* if given any arbitrary open covering of X there exists a finite subcover that still covers X. As compactness is a topological property we have the following result as well

Theorem: *Let $f : X \to Y$ be continuous. If X is compact, then so is $f(X)$.*

So one immediate consequence is that all finite sets are necessarily compact if you take an open ball around each point you can obtain the finite subcover. Additionally, another less obvious example is that when restricted to the space of real numbers, this notion of compactness is actually equivalent to a set being both closed and bounded. One of the most famous and widely used closed and bounded sets of real numbers happens to be the closed interval $[a, b]$. As a matter of fact this result is so widely used it even has a name, it is called the Heine-Borel Theorem which states

> Theorem (Heine-Borel): *Any subset of* $\mathbb{R}$ *is compact if and only if it is closed and bounded.*

Thus one immediate result is that the reals themselves are not compact as they are closed but not bounded. So suppose then that f is some continuous function defined over the interval $[a, b]$, (which is actually something that appears in other math courses such as optimization or Nonlinear dynamics and chaos) then by the continuity of f, the set $f([a, b])$ is compact and all compact sets have a min and max giving us the following result

> Theorem: *Continuous functions over compact sets attain their min and max points*

Which I learned how to prove in my point set topology course.

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Part §II.3 Complex Analysis I

In this course we extended the notions of continuity and differentiablity from Real Analysis to the (field of) Complex numbers. We started off with some of the fundementals of the Complex numbers such as the additive and multiplicative structure of $\mathbb{C}$. More precisely, we define $\mathbb{C}$ to be the set of all ordered pairs (a, b) where $a.b \in \mathbb{R}$. Then we can define addition and multiplication on $\mathbb{C}$ as follows

$$\begin{aligned} (a,b)+(c,d) &= (a+c, b+d) \\ (a,b)\cdot(c,d) &= (ac-bd, bc+ad). \end{aligned}$$

In fact, one can easily verify $\mathbb{C}$ is a field with the above operations defined. One immediate result is the fact that $i^2 = -1$ and I never understood why this was true or where it even came from. This can be proven if we take $i = (0, 1) \in \mathbb{C}$, then compute $(0, 1) \cdot (0, 1)$ as above, which then reduces to -1. Next we covered the representation of complex numbers into real and imaginary parts and some properties of the modulus which we define for a given complex number $z = x + iy$ where $x, y \in \mathbb{R}$, as

$$|z| = \sqrt{x^2 + y^2},$$

and we can denote the real and imaginary parts as $\mathrm{Re}(z)$ and $\mathrm{Im}(z)$ respectively. So if $z = 4+3i$, then $\mathrm{Re}(z)$=4, $\mathrm{Im}(z)$= 3. Furthermore, we can take on the polar representations $x = r\cos\theta, y = r\sin\theta$ we can represent a complex number z as $r(\cos\theta + i\sin\theta)$, where $\theta \in \mathbb{R}$ and $r = |z|$ then the polar coordinate is given by (r, θ).

Next we moved into some actual analysis starting off with convergence of power series which is a motivating factor behind Complex analysis. We examined certain properties from Real analysis that carry over nicely such as the convergence of the trig and exponential functions. However the notion of continuity for a real valued function is much nicer in some sense than that of continuity for a complex valued function. Given a real valued function f, there is a nice concise test for the limit existing at some point $x_0 \in \mathbb{R}$. That is, if both

$$\lim_{x \to x_0^-} f(x)$$

and

$$\lim_{x \to x_0^+} f(x)$$

exist and are equal, then

$$\lim_{x \to x_0} f(x)$$

exists. However for a given complex valued function $f(z)$, $z \in \mathbb{C}$, we have an uncountable number of sides for a fixed $z_0 \in \mathbb{C}$ to consider thus a mere left and right sided limit test would no longer suffice. And so $complex\ differentiability$ is stronger than $real\ differentiability$. Let $G \subseteq \mathbb{C}$ be an open subset. We say a complex valued function $f : G \to \mathbb{C}$ is differentiable at a point $z_0 \in G$ if

$$\lim_{\Delta \to 0} \frac{f(z_0 + \Delta) - f(z_0)}{\Delta}$$

exists (and so if f is differentiable at every point of its domain we say just f is differentiable and drop the fixed point) and thus $f' : G \to \mathbb{C}$ defines a new function and if f' is continuous, we say that f is continuously differentiable or simply *analytic*. More precisely, if a complex-valued function is given by it's power series

$$f(z) = \Sigma_{n=1}^{\infty} a_n (z-a)^n$$

with a radius of convergence $R > 0$, then its kth derivative is given by

$$\Sigma_{n=k}^{\infty} n(n-1) \cdot ... \cdot (n-k+1) a_n (z-a)^{n-k}$$

and has radius of convergence R as well and $f \in C^{\infty}$. Thus any complex valued function that has a continuous first derivative automatically admits a continuous derivative for each k which is one of the many beauties of the study of complex variables. Consequently, any power series with radius of convergence $R > 0$ is the series of an analytic function defined on an open radius R ball in $\mathbb{C}$. Thus one can show that since the radius of convergence of

$$\Sigma_{n=0}^{\infty} \frac{z^n}{n!}$$

is ∞, then the exponential function e is analytic on the entire complex plane. We concluded this introductory section with a result about constant analytic functions defined over open and connected subsets of the complex plane, which happen to in fact be topological properties.

> Theorem: *If $G \subseteq \mathbb{C}$ is open and connected and $f : G \to \mathbb{C}$ is analytic with $f'(z) = 0$ for every $z \in G$, then f is constant on all of G.*

Another very useful test for telling when a complex-valued function is constant is known as the Maximum Modulus Principle which again, pulls from topology to prove and it states:

> Theorem (Maximum Modulus): *Let $G \subseteq \mathbb{C}$ be open and connected and $f : G \to \mathbb{C}$ analytic. If there exists a point $a \in G$ such that*
>
> $$|f(a)| \geq |f(z)|$$
>
> *for every $z \in G$, then f is constant.*

Next we focus our attention on the Cauchy Riemann equations. Let $G \subseteq \mathbb{C}$ be open and connected and

$$f : G \to \mathbb{C}$$

be continuous. As z can be given by $x + iy$ where $x, y \in \mathbb{R}$, we can write $f(z) = f(x + iy)$. Then as mentioned above, we can write

$$u(x, y) = \text{Re}[f(x + iy)], v(x, y) = \text{Im}[f(x + iy)].$$

Here u, v are both real valued functions from $\mathbb{R}^2$ into $\mathbb{R}$. If we take the derivative of $f(z)$ using real and imginary parts we end up with some partial differential equations knows famously as the Cauchy-Riemann equations. So if f is a complex-valued function defined on some open and connected subset of $\mathbb{C}$ where we can write $f(x+iy) = u(x,y) + iv(x,y)$, we say that f satisfies the Cauchy-Riemann equations if the following hold for the partial derivatives of u and v:

$$u_x = v_y$$

$$u_y = -v_x.$$

One major result of functions that satisfy the Cauchy-Riemann equations is that the are in fact analytic on their entire domain on which the function is defined. In fact, we will see shortly that a given complex-valued function is analytic if and only if it satisfies the Cauchy-Riemann equations. For example, one can verify the function $f(z) = z^2$ defined on all of $\mathbb{C}$ satisfies the Cauchy-Riemann equations (and is therefore analytic as well on all of $\mathbb{C}$). Let $z = x + iy$, then

$$\begin{aligned} f(z) &= z^2 \\ &= (x+iy)^2 \\ &= x^2 + ixy + ixy + i^2y^2 \\ &= (x^2 - y^2) + i(2xy) \end{aligned}$$

Then the real and imaginary parts are given via

$$u(x,y) = x^2 - y^2$$

and

$$v(x,y) = 2xy$$

respectively. Then

$$\begin{aligned} u_x &= 2x \\ &= v_y \end{aligned}$$

And

$$\begin{aligned} u_y &= -2y \\ &= -v_x \end{aligned}$$

Thus $f(z) = z^2$ satisfies the Cauchy-Riemann equations. This leads us to another major result which is more of a test for analyticity that states

> Theorem: *Let u, v be real-valued functions defined on an open connected set G suppose u, v have continuous partial derivatives. Then $f : G \to \mathbb{C}$ is analytic iff u, v satisfy the Cauchy-Riemann equations.*

Next we move on to complex integration which was the key focus of the course.

Before stating Cauchy's Integral formulas, we must define line integrals. If $\gamma : [a, b] \to \mathbb{C}$ is a curve and f is a function defined and continuous on the range of γ, then the line integral of f along γ is given by

$$\int_a^b f(\gamma(t))\gamma'(t)d(t).$$

One immediate consequence of this is the case where our path γ is closed, that is $\gamma(a) = \gamma(b)$. In this special case the line integral of f over γ is zero assuming f is analytic on a disk that γ lies in. This allows us to state Cauchys Integral formula. Before we proceed we must give one more definition, for any path γ and point in the domain, say a define $\eta(\gamma; a) \in \mathbb{Z}$ as the winding number of γ around a, that is, the number of times γ 'wraps' around a. Now for Cauchys big integral formula:

> Theorem: *Let $G \subseteq \mathbb{C}$ be open and $f : G \to \mathbb{C}$ analytic. If γ is a closed curve in G such*

that the winding number is 0 for every point in G, then for any $a \in G \backslash \gamma([a,b])$, we have that

$$\eta(\gamma; a) f(a) = \frac{1}{2\pi} \int_{\gamma} \frac{f(z)}{z-a} dz$$

And Cauchy's Theorem which states:

Theorem: *Let $G \subseteq \mathbb{C}$ be open and $f : G \to \mathbb{C}$ analytic. If $\gamma_1, ..., \gamma_n$ are all closed curves in G such that the sum of their winding numbers is 0 for every $w \in \mathbb{C} \backslash G$, then*

$$\Sigma_{i=1}^{n} \int_{\gamma_i} f = 0.$$

One thing I wondered myself when I first took Calculus was how do we integrate functions that are complex valued and it turns out this course gives you the appropriate tools for this. I will end the section on Introduction to Complex Analysis with a big punchline which states if a given complex-valued function is analytic, then it necessarily admits a power series! Or more formally:

Theorem: *Let f be analytic on the ball $B_R(a)$. Then f has a power series, namely,*

$$f(z) = \Sigma_{n=0}^{\infty} a_n (z-a)^n,$$

for $|z-a| < R$. Here the coefficients are given as

$$a_n = \frac{1}{n!} f^{(n)}(a)$$

and this series has radius of convergence $\geq R$.

This concludes my introduction to the study of complex variables.

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Part §III.1 Tie it together

In summary, I would like to give a brief analysis of my degree, both implicit and explicitly. First I will go over ties and connections made between different fields of mathematics which is more of an explicit analysis of the courses. Next I will go over the challenges and obstacles I have faced both as a learner and as a doer of mathemtics. In this latter analysis, which is the more implicit one, I will go over two key challenges that I faced: Failure is flattering, trust your gut.

Often times in mathematics we are concerned with which properties from certain fields can be extended to be applied to another field. We end up with fields such as Algebraic Topology, Differential Geometry, or Algebraic Geometry or I like to call them "superfields". I have witnessed a bit of this after having taken these courses. I myself have been able to make several myself. For instance if we were to dig deeper into topology we define what it means for two given paths to be "the same" or equivalent, we call this notion of equivalence a *path homotopy.* Then one could reformulate Cauchy's Theorem not in terms of total winding number being 0 anymore but rather in terms of the paths being *homotopic* to the trivial or the 0 path. We have seen on so many more occasions that topology is used as a tool kit course in aiding analysis and algebra. Another place we can see this is in the proof of the theorem at the top of page 16, in proving the function f is constant on all of G, we appeal strictly to a mere topological argument of what it really means for a space to be connected. Moreover, topology has been a useful tool in producing new proofs to existing solutions. One can use this notion of open and closed sets alone to prove a fact from number theory which can be proven independently of the number theoretic proof of the fact that there are infinitely many prime numbers.

Similarly one could use number theory and facts about prime divisors to actually construct a totally new metric on the integers (and even extend this to the rationals) which turns out to be useful for the analysis of $\mathbb{Z}$ and $\mathbb{Q}$. In fact the integers are a space that cannot be covered by a finite number of epsilon balls in the standard Euclidean metric (since under the standard absolute value distance/metric elements of $\mathbb{Z}$ get arbitrarily far away from one another). However under this new metric defined using number theory (the p-adic metric) we can now totally bound the integers!

That is, cover all integers with a finite number of epsilon balls! These are just a few examples of all the possible ties amongst distinct disciplines of mathematics.

Lastly I will analyze my time here implicitly as a learner and a doer of mathematics. One of the two I like to refer to as ”Failure is flattering” as apposed to it not being flattering. In that, I refer to the moments we come across big failures which in the long run we only learn and grow from as apposed to the failure holding you back in life. Not only has my time in this program been overdue but towards the end when I finally decided to fight off my inner demons I came back sober from an ongoing drug addiction and my very first semester back had two very big setbacks: I got an F in my first course back and failed both my comprehensive exams. Now at this point my GPA had already dropped to an all time low and I had dropped in and out taking various tutoring jobs to only realize this is all I am good for. I used to take failure as a sign to stop and turn the other direction. However as of lately I have grown to learn that these are instances for me to learn from, for example one particular problem I got incorrect on the comprehensive for Topology was that I assumed the term ”separable” meant a space had a separation when in fact it meant the space had a countable dense subset, two very distinct definitions. I learned to use my failures not as set backs but rather learning experiences for the next time.

Lastly, I have always struggled with trusting my inner gut when it comes to doing mathematics (specially in the case I am alone at the moment doing math). Time after time again in this program I have been brought to realize that my own gut intuition is more often than not the right way to go. I often times doubt my own solutions and I freeze up in the moment rather than run with my gut. What this in turn does for me is gives me more of a trial and error approach which again comes with certain failures that I no longer let get in my way but rather work past them and persevere during times of uncertainty. I learned to trust myself and just do it!

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