Free Algebra Supplement

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2021

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Properties of $\mathbb{R}$

Here are the properties of the real numbers. For all real numbers a, b, and c the following rules are always true:

$a(b+c) = ab + ac$	distributive law
$a + b = b + a$	commutative law of +
$a \cdot b = b \cdot a$	commutative law of $\cdot$
$a + (b + c) = (a + b) + c$	associative law of +
$a(bc) = (ab)c$	associative law of $\cdot$
$a + 0 = a$	additive identity
$(a)(1) = a$	multiplicative identity
$a + (-a) = 0$	additive inverse
$a(\frac{1}{a}) = 1$	multiplicative inverse

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Exponents

For any real number x and any integers a, b (of course supposing $b \neq 0$) the following always hold:

1. $x^{\frac{a}{b}} = \sqrt[b]{x^a} = (\sqrt[b]{x})^a$; so then $x^{\frac{1}{n}}$ is the nth root of x, $\sqrt[n]{x}$.
2. $(x^a)^b = x^{a \cdot b}$; here the powers multiply
3. $x^a x^b = x^{a+b}$; here the powers add
4. $x^0 = 1$; anything to the 0 power ia 1
5. $x^1 = x$; anything to the 1 power is itself
6. $x^{-a} = \frac{1}{x^a}$
7. $x^a = \frac{1}{x^{-a}}$

For 6. and 7. a can be rational, i.e., a fraction as well.

Some Examples of these being used

- $(2^2)^2 = 2^{2 \cdot 2} = 2^4 = 16$
- $8^{\frac{2}{3}} = (\sqrt[3]{8})^2 = (\sqrt[3]{2 \cdot 2 \cdot 2})^2 = 2^2 = 4$
- $9^{\frac{-3}{2}} = \frac{1}{9^{\frac{3}{2}}} = \frac{1}{(\sqrt{9})^3} = \frac{1}{3^3} = \frac{1}{27}$

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Perimeter, Area , & Volume

Perimeter

1. $P_{\text{rectangle}} = 2l + 2w$; l is length, w is width.
2. $P_{\text{square}} = 4s$; s is a side length
3. $P_{\text{circle}} = 2\pi r = \pi d$; this is also known as a circumference, r is radius and d is diameter.

Area

1. $A_{\text{rectangle}} = lw$
2. $A_{\text{square}} = s^2$
3. $A_{\text{circle}} = \pi r^2$
4. $A_{\text{triangle}} = \frac{1}{2}bh$; b is base length, h is height

Volume

1. $A_{\text{box}} = lwh$; h is height here.
2. $A_{\text{sphere}} = \frac{4}{3}\pi r^3$
3. $A_{\text{cone}} = \frac{1}{3}\pi r^2 h$
4. $A_{\text{cylinder}} = \pi r^2 h$

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4. Lines and slope intercept form

So a *line* is a collection of points $\{(x, y)\}$ so that they satisfy the equation

$$y = mx + b$$

where m is slope and b is the y-intercept. That is, the line ALWAYS has a point on the graph at the point $(0, b)$ with slope m.
Given two points $(x_1, y_1), (x_2, y_2)$ on the $xy-$plane the *slope* os the line passing through them is given by the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

So if they give you two points and ask for the equation of the line between them:

- **First**, find the slope m.
- **Second**, pick (any) one of the two given points and plug these in for x and y and you plug m in as well into:

  $$y = mx + b$$

  and you can find b this way!
- **Third**, write the $y = mx + b$ formula by only plugging in for the m and the b which is final answer.

For example if you are given points $(-2, -2)$ and $(1, 4)$ we **First** find m:

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4 - (-2)}{1 - (-2)} \\ &= \frac{4 + 2}{1 + 2} \\ &= \frac{6}{3} \\ &= 2. \end{aligned}$$

Second, I pick the point $(1, 4)$ and we use this together with $m = 2$ to find b:

$$(4) = (2)(1) + b$$

And so $b = 2$. So **Third** we have

$$y = 2x + 2$$

as our equation.

More examples with answers for you to try in the Exercises: Algebra section!

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5. Distance VS. Mid-Point

Whenever you are given two points

$$(x_1, y_1), (x_2, y_2),$$

and are not concerned with the line between them there's two other definitions you can find using them:

1. Distance
2. Mid-Point

The key difference is that distance take in two points and gives you a single number that is never negative. Mid-Point on the other hand give you a new POINT! And the components could be negative or zero or the same!

Given two points

$$(x_1, y_1), (x_2, y_2),$$

their *distance* is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

On the other hand, given two points

$$(x_1, y_1), (x_2, y_2),$$

their *mid-point* is given by

$$\text{m.p.} = (\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}).$$

<u>Example:</u> For the points $(3, 2)$,$(-1, 5)$ we find both distance and mid-point.

For distance we have

$$\begin{aligned} d &= \sqrt{(-1-3)^2 + (5-2)^2} \\ &= \sqrt{(-4)^2 + (3)^2} \\ &= \sqrt{16+9} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

For mid-point we have

$$\begin{aligned} \text{m.p.} &= (\frac{3+(-1)}{2}, \frac{2+5}{2}) \\ &= (\frac{2}{2}, \frac{7}{2}) \\ &= (1, 3.5). \end{aligned}$$

So you see, the answers look so different.
More examples with answers for you to try in the Exercises: Algebra section!
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6. Systems of Equations

Anytime you are given two equations and two variables, usually x and y, we refer to the combination of both as a **system of equations**. And there are three methods of solving, two of which we will cover here, the third is merely graphing both lines.

Given a system of equations say

$$\begin{cases} y = 2x - 1 \\ 2x - y = 4 \end{cases}$$

The two ways of solving would be referred to as the **substitution** method and **elimination**. Typically when one of the two given equations has solved for x or y already substitution is the way as with the above example.

Substitution So we are given the system of equations

$$\begin{cases} y = 3x - 1 \\ 2x - y = 4 \end{cases}$$

As you can see, the top equation has y isolated so we can take that y and plug it into the second equation! And lets not forget the negative, so I recommend starting with parenthesis around what you *substitute* in:

$$\begin{aligned} 2x - (3x - 1) &= 4 && \text{plugging in } y = 3x - 1 \text{ into the second equation} \\ 2x - 3x + 1 &= 4 && \text{by distributing the negative} \\ -x &= 3 && \text{collecting together like terms} \\ x &= -3 && \text{divide by } -1 \end{aligned}$$

So we now have $x = -3$ and to find y we plug this into either original equation, let us use the first one:

$$\begin{aligned} y &= 3(-3) - 1 \\ &= -9 - 1 \\ &= -10 \end{aligned}$$

So $y = -10$ and the solution is written as $(-3, -10)$.
Exercise: Switch the first one from $y = 3x - 1$ to $y = 2x - 1$ and see what happens!

Elimination For elimination I give an example of a system that is more "set up" for elimination. By this I mean, some systems are easier done via substitution and other via elimination thus we examine both!
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7. Properties of the Natural log

1. $\log_a(x) + \log_a(y) = \log_a(xy)$
2. $\log_a(x) - \log_a(y) = \log_a(\frac{x}{y})$
3. $\ln(x) + \ln(y) = \ln(xy)$
4. $\ln(x) - \ln(y) = \ln(\frac{x}{y})$
5. $\log_a x^n = n \log_a x$
6. $\log_a x = \frac{1}{\log_x a}$
7. $\log_a 1 = 0$
8. $\log_a a = 1$
9. $\log_{a^m} a^n = \frac{n}{m}$
10. $\ln x^n = n \cdot \ln x$
11. $\ln e = 1$
12. $\ln 1 = 0$
13. $\log_e x = \ln x$
14. $\log_a x = b$ means $a^b = x$.
15. $e^{\ln(\text{stuff})} = \text{stuff}$
16. $\ln e^{(\text{stuff})} = \text{stuff}$

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8. Radical Expressions

Whenever solving a radical expression like

$$\sqrt[n]{\text{some stuff}}$$

The point is to write the stuff inside in a way to get things times themselves n times, for example, if $n = 3$ and the stuff inside is $8x^3y^4$ then we have

$$\sqrt[3]{8x^3y^4} = \sqrt[3]{2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y}$$

And so we can pull out exactly ONE 2, exactly ONE x, and exactly ONE y with one single y remaining underneath the cubed root and so the answer becomes just

$$2xy\sqrt[3]{y}.$$

Another examplem when there is no number it is automatically a 2, for example:

$$\sqrt{27a^3b^4c}$$

can be written as

$$\sqrt{3 \cdot 3 \cdot 3 \cdot a \cdot a \cdot a \cdot b^2 \cdot b^2 \cdot c}$$

So we can pull out exactly ONE 3, exactly ONE a, exactly ONE b^2 and what is left inside is a $3ac$, so the answer just becomes

$$3ab^2\sqrt{3ac}$$

More examples with answers for you to try in the Exercises: Algebra section!

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Compound Interest Formulas

There are two main formulas:

Compound continuously is given by

$$A = Pe^{rt}$$

Where

$$\begin{aligned} A; &\quad \text{end amount} \\ P; &\quad \text{initial amount} \\ r; &\quad \text{rate as decimal} \\ t; &\quad \text{total amount of time} \end{aligned}$$

And

Compound Interest monthly,weekly,quarterly, annually, daily given by

$$A = P(1 + \frac{r}{n})^{nt}$$

Where

$$\begin{aligned} A; &\quad \text{end amount} \\ P; &\quad \text{initial amount} \\ r; &\quad \text{rate as decimal} \\ t; &\quad \text{total amount of time} \\ n; &\quad \text{number of times per year.} \end{aligned}$$

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